Carrots and Sticks: Punishment and Party Power in Congress*

Erik Snowberg
Stanford GSB
snowberg@stanford.edu
www.stanford.edu/~esnowber

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Abstract

This paper proposes a dual-utility theory of parties in a legislature. In this theory a legislator has preferences over both actions and policy outcomes. Specifically, a legislator's utility is determined by *position taking*—his own votes—and by *partisan utility* which depends on policy implemented by the legislature. Party leaders design mechanisms that make legislators better off by co-ordinating votes and compensating those legislators that vote against the interests of their constitutents. The model produces two main findings. First, party leaders are more likely to use promises of rewards and threats of punishment as the size of the party or the benefit of passing the party's policy platform increases. Secondly, and perhaps counter-intuitively, party leaders become less likely to use rewards and punishments when the number of centrist legislators increases, or the costs to centrist legislators increase.

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Tom Delay and Carl Albert were both leaders of the majority party in the U.S. House of Representatives. Beyond an occupation and similarly limited physical stature, these two men had very little in common. Delay was a master of threatening and cajoling majority party members into voting for the party's policy platform. He doled out record amounts of campaign funds to loyal party members through his political action committees and associated lobbyists. He coordinated schemes to transfer funds between candidates. His less pleasant side was exposed by his habit of threatening to run a primary challenger against disobedient legislators (Dubose and Reid, 2004). Albert, on the other hand, has been characterized as "inactive and weak."[1]

Aside from the men themselves, the caucus rules governing the conduct of party leaders were vastly different during the times when Delay and Albert were party leaders. Albert lead during a time when caucus rules and norms enhanced the standing of committee and subcommittee chairs at the expense of party leaders. When Delay entered the leadership, his party caucus approved rules that increased the role of party leaders in selecting and maintaining committee chairs and committee membership, along with other reforms that enhanced the standing of party leadership.

This paper examines two interrelated questions suggested by the previous paragraphs. First, when will legislators choose leaders—and rules that will allow leaders—to use threats of punishment or the promise of rewards to pass policy? Second, if legislators wish their party leaders to take an active role, under what conditions will they prefer a regime that focuses on the use of threats of punishment to a regime that uses the promise of rewards?

The dual-utility model presented in this paper makes two main assumptions. First, party leaders in Congress, and the rules that govern their behavior, are selected by party legislators. Second, departing from the standard spatial model, legislators derive benefit from two sources. First, legislators have preferences over their own actions: *position-taking*

[1]The quote is from Cox and McCubbins (1993, p. 156). See also Peters (1997, pp. 174-175). Albert did not see himself as particularly ineffective, although there is little data to defend his position (Albert and Goble, 1990).

utility derives from the electoral consequences of voting for or against the party's policy platform. Second, legislators have preferences over policy outcomes: *partisan utility* comes from the policy consequences of passing or defeating the party's policy platform.

Within the party analyzed here legislators are assumed to receive the same, positive, partisan utility when the party's policy platform passes. There are thus two types of legislators who are distinguished by whether their position-taking utility—their preferences over actions—reinforces or countervails their partisan utility—their preferences over policy outcomes. While all party legislators would like to see the party's platform pass, *centrist* legislators suffer an electoral cost if they vote for the party's policy platform, while *stalwart* legislators gain an additional electoral benefit from the same action.

Party leaders can increase the expected utility of all legislators by implementing an institution or *mechanism*. Yet centrist and stalwart legislators will have different preferences over the type of mechanism. Centrists prefer a *carrot regime* which compensates them for the electoral costs of voting for the party's policy platform. Stalwarts prefer a *stick regime* where centrist legislators are threatened into voting for the party's platform at no cost to the stalwarts. The mechanism in each regime is thus designed to attract the support of the group that has a natural preference for the other regime: the mechanism in the carrot regime is designed to maximize the support of that regime among stalwarts, while the mechanism in the stick regime is designed to maximize support among centrist legislators. Even so, both mechanisms may fail to garner enough support, and legislators may opt instead to forgo partisan utility by selecting *non-delegation* in which each legislator votes according to his position-taking utility.

The paper proceeds as follows. The basics of the model are first described, and the principles used to find the optimal mechanism under each regime are discussed. An optimal mechanism under each regime is then designed. Finally, the model is used to understand how changes in the parameters of the model affect the probability that one regime or the other (or non-delegation) are adopted by the party caucus.

There are two main findings from the dual-utility model. The first is that party leaders are more likely to use promises of rewards or threats of punishment as the size of the party (rather than non-delegation) or the magnitude of partisan utility increases. The second result is that, perhaps counter-intuitively, party leaders become less likely to use either rewards and punishments when the number of centrist legislators increases, or the electoral costs to centrist legislators increase. Why do party leaders fail to play a role precisely when they have the most work to do? The answer is that when centrist legislators have higher costs, they must be compensated at higher rates (or lose more when threatened) making it less likely that legislators will gain by paying to pass the party's policy platform. Legislators look forward to this outcome, and choose not to empower party leaders to offer threats and rewards.

1.1 Relationship to the Literature

In recent years congressional scholars have fought to a stand-still over the (non-)importance of party leaders to legislative outcomes in the U.S. Congress. Both literatures start from the characterization of legislators as "single-minded seekers of reelection" (Mayhew, 1974, p. 5). However, these literatures rely on different assumptions about what party leaders have to offer legislators.

The theoretical treatments that ignore party leaders assume that whatever party leaders have to offer legislators must originate from the legislators themselves. If legislators only value re-election, and gain re-election only through taking votes preferred by their constituents, party leaders can only be of use to legislators if they have superior knowledge of a legislator's district. This is unlikely to be the case. That is, if party leaders have nothing to give to legislators, legislators will not empower party leaders.[2]

The opposing literature thus has a clear starting point; it assumes that party leaders have something to give legislators to help them with their electoral pursuits. This something

[2]This is formally discussed in Krehbiel (2006, 2007) and provides a rationale for not explicitly modeling party in Krehbiel (1991, 1998).

varies, but it is usually given exogenously to party leaders. As this something is given exogenously at no cost to party leaders, it doesn't depend on party leader's actions, and thus the preferences of party leaders alone determine legislative outcomes. The conclusions of these models are thus driven by assumptions about the preferences and constraints of party leaders (Cox and McCubbins, 1993, ch. 5).

The dual-utility theory presented here builds on both of these approaches: party leaders have a role in generating policy outcomes preferred by legislators, but they do so at the discretion of party legislators. As such, the theory follows the conditional party government theory of Aldrich and Rohde (Rohde, 1991; Aldrich, 1995; Aldrich and Rohde, 1997). This implies the dual-utility theory has some similarities to party cartel theory. The approach and substantive question addressed here are different from those in party cartel theory, the basic assumptions of the models differ in important ways as well.[3]

First, in party cartel theory legislators are assumed to vote against their districts' interests because doing so improves the public perception of the party, and thus improves legislators' reelection prospects (Cox and McCubbins, 1993, p. 110).[4] This implies, paradoxically, that voting yes on a bill damages a legislator's chance of re-election, while the passage of the same bill results in a larger increase in the same legislator's chance of re-election. In contrast, in the dual-utility theory presented here, legislators will take votes that decrease their chance of reelection because it increases their overall utility, not their chance of re-election. The implicit characterization of party here is thus closest to Burke (1784, p. 110): "Party is a body of men united, for promoting ... some particular principle in which they are all agreed upon."

Second, a legislator's benefit from good public perception of his party depends on the ability of the majority party in the legislature to change national circumstances. Cox and

[3]The party cartel theory of Cox and McCubbins (1993) is further developed in Cox and McCubbins (2002) and Cox and McCubbins (2005), but the portions of the theory developed in the latter two works are less relevant to this paper.

[4]Party-cartel theory leaves open the possibility that legislators are not single minded seekers of reelection. However, it does not address what other motivations might be possible, and how these may change the implications of the theory.

McCubbins specifically point to the economy as a realm where the majority party has an effect that changes the electoral chances of their candidates. Yet recent research shows that the ability of congressional parties to change economic aggregates is roughly one-tenth that of the president (Snowberg, Wolfers and Zitzewitz, 2007*a*,*b*). As Cox and McCubbins (1993, p. 121) note these facts make "the prospects for [their own] argument ... bleak."

This paper is most similar to Patty (2006) which also assumes legislators receive separate utility from their votes and from the policy implemented. At a technical level Patty differs in that party leaders can only use a uniform (rather than discriminatory) punishment to induce votes from legislators, and that this level of punishment is decided before legislators know their position-taking utilities. Substantively, Patty concludes that party strength is decreasing in majority party size, in stark contrast with the model in this paper. Interestingly, the results of his empirical test are consonant both with his model and the model in this paper, which have opposite conclusions regarding the relationship between size and party strength, casting doubt on the validity of his test.[5]

Vote buying has been studied in a large literature. Snyder (1991) provides a clean model which is extended to consider two vote buyers in Groseclose and Snyder (1996). Many of these models are specific to the changing of policy through campaign finance or lobbying, for example: Becker (1983), Prat (2002) and Coate (2004).

Punishment is studied less often. The first examination of punishment in an electoral environment is contained in a section of Baron (1989). More recent papers by Dal Bó and Di Tella (2003), Dal Bó, Dal Bó and Di Tella (2006, 2007), Dal Bó (2007) study the equilibrium effects of threats on the quality of politicians and policy.

[5]For other models of equilibrium party strength in the tradition of conditional party government see Iaryczower (2008) and Volden and Bergman (2006). Ashworth and Bueno de Mesquita (2004) use a dual-utility framework to examine party strength in parliamentary and presidential democracies. A subsection of Dal Bó (2007) also uses a dual-utility framework in discussing vote buying and punishment.

2 The Model

2.1 Legislators

A legislature $\mathcal{L}$ is composed of an odd number of legislators i who are either members of the majority party $i \in \mathcal{M} \subset \mathcal{L}$ or the minority party $i \in \mathcal{L} \setminus \mathcal{M}$. The legislature decides whether to adopt or reject the majority party's *policy platform* via majority rule.[6] The majority party has $m = |\mathcal{M}|$ members. True to its name, the majority party comprises over half of the legislature: $\frac{m}{|\mathcal{L}|} > \frac{1}{2}$. The majority party will be the focus of analysis and will be referred to as simply the party from now on.

All party legislators $i \in \mathcal{M}$ receive *partisan utility* of B if the majority party's platform passes. Individual legislators have different *position-taking utilities* that depend on their vote on the party's policy platform and their constituent's preferences. There are $n < m$ *centrist* legislators whose constituents oppose the party's platform. These legislators suffer a position-taking cost $-v_i^c < 0$ if they vote for the party's platform.[7] The remaining $m - n$ legislators are stalwarts—their constituents support the party's platform. Stalwarts suffer a position-taking cost $-v_i^s < 0$ if they vote against the party's platform. The payoffs to legislators for each action in each outcome are detailed in Table 1.

Although it is common knowledge whether a legislator is a centrist or an stalwart, the magnitude of the position-taking utility of legislator i is private information. The position-taking cost v_i^c to a centrist legislator of voting for the party's policy platform is independently drawn from a distribution with probability density function $f(v)$ with cumulative distribution function $F(v)$. This distribution is assumed to be continuous, atomless and to have full support on the interval $[0, \overline{v}]$. Stalwart legislators will always vote for the party's platform, so their position-taking utilities are unimportant to the analysis. For the party's platform

[6] This model is easily applied to individual bills or policies. However, for consistancy with the motivating question, which compares leaders' behavior between Congresses (rather than within a single Congress) the model focuses on the set of all policies, or the party's policy platform.

[7] More explicitly, centrist legislator's constituent's prefer the (unmodeled) status quo to the party's platform.

Table 1: A legislator's payoffs depend on whether or not they voted for the platform, and whether or not the platform passes.

Legislator i's Vote	Platform Passes?	Centrist's Payoff	Stalwart's Payoff
Yes	Yes	$B - v_i^c$	B
	No	$-v_i^c$	0
No	Yes	B	$B - v_i^s$
	No	0	$-v_i^s$

to pass, $k = \frac{|\mathcal{L}|+1}{2} - (m - n)$ centrist legislators must vote for it, where k is assumed to be greater than zero.

2.2 Party Leaders

In the dual-utility model, party leaders are automatons that follow some combination of rules laid down for them by party legislators and the dictates of their own internal rules. Leadership elections are thus a problem in mechanism design, where legislators select rules and personalities that maximize their expected payoffs.[8]

Legislators consider mechanisms from three regimes:

- In the *carrot regime* party leaders can credibly promise to transfer utility from stalwart legislators to centrist legislators to encourage centrist legislators to vote for the party's platform.

- In the *stick regime* party leaders can credibly threaten to impose a cost on centrist

[8]The model makes no distinction as to whether constraints on a leader's behavior are externally imposed through rules, or internally imposed by a leader's personality or character.

legislators if they vote against the party's platform.

- In the third regime, called *non-delegation*, party leaders are not empowered to do anything in order to pass the party's platform.

Legislators select a mechanism for their leaders to implement. Choosing a mechanism is the same as choosing a schedule of payments or punishments that is contingent on the bids b submitted by and the votes of *all* centrist legislators. In the carrot regime this schedule can contain only non-negative payments to centrist legislators (note that zero is a non-negative payment). In a stick regime, by contrast, the schedule can contain only punishments (or costs) to centrist legislators (note again the schedule can include zeros).

Once the mechanism is chosen, each legislator i submits a bid b_i. Legislators then observe all bids. Each legislator votes and payments or punishments are delivered according to the schedule promised by party leaders.[9]

Some of these steps are ignored in standard mechanism design. For example, if centrist legislators are paid their bid if they vote for the party's platform, then no legislator will submit a bid that is less than his cost of voting for the party's platform. As such, each legislator will vote for the party's platform as this will give him higher utility than voting against it. The steps after a legislator bids are often ignored as the decisions and actions thereafter are automatic. However, as will be seen, these additional steps are less intuitive in the stick regime.

Informally, the sequence of events in the model is:

1. Nature picks the party's platform, and the position-taking utilities of each legislator.

2. Each legislator calculates his expected utility under each of the three regimes.

[9]Define the vote of legislator i as $\sigma_i \in \{0,1\}$ where $\sigma_i = 1$ represents legislator i voted for the party's platform. Further, define b as the vector of all centrist legislator bids, and σ as the vector of all centrist legislator votes. Then, formally a mechanism is a pair $(\mathcal{A}(b), \mathcal{P}(b,\sigma))$ where $\mathcal{A}$ is the allocation rule, and $\mathcal{P}$ is the payment rule. The allocation rule $\mathcal{A} : \Re^n \to \{0,1\}^n$ specifies which legislators will vote for the platform based on the bids of all centrist legislators. The payment rule $\mathcal{P} : \Re^n \times \{0,1\}^n \to \Re^n$ specifies how much each centrist legislator should pay or be paid based on the vector of bids and the votes of all centrist legislators.

3. Majority party legislators (collectively) decide which regime and mechanism to select.

4. Party leaders attempt to pass the party's platform using the mechanism designed by legislators.

5. The party's platform is voted on using majority rule, and payoffs are realized.

Three assumptions are maintained throughout the paper.

Assumption 1 (No Collusion) *Centrist legislators cannot transfer utility between themselves, except indirectly through party leaders.*

Assumption 2 (Commitment) *If party leaders commit to a reward or punishment contingent on some action, and this action is taken, then the reward or punishment is delivered with certainty, subject to budget constraints.*

Assumption 3 (Sticks are Feasible) *The magnitude of the disutility of a centrist legislator from voting for the party's platform v_i^c is bounded above by the total utlitiy accruing to legislators if the platform passes: $\overline{v} < mB$.*

The first assumption prohibits centrist legislators from forming their own party within the party for the purpose of manipulating mechanisms or extorting stalwart legislators. Theoretically, this assumption allows me to ignore commitment issues within sub-party organizations. Commitment issues still exist for the broader party, however. As party leaders do not reward or punish legislators until the end of the game, party leaders would always prefer not to incur the costs associate with these actions. Legislators, aware that promises and threats are not credible would only vote based on their position-taking utility. In order to ensure more interesting equilibria, Assumption 2 assumes that party leaders can commit to reward or punish legislators based on their actions.[10] The third assumption guarantees

[10]The existence of real world commitment mechanisms for rewards has been examined by Weingast and Marshall (1988). Commitment to punishments is sustainable in repeated games (Dal Bó, Dal Bó and Di Tella, 2007). In the dual-utility model, commitment is more likely to come from electing a party leader with a vindictive personality, such as Delay.

that party leaders can use threats of punishment to pass the party's platform, as will be seen in Section 4.

2.3 Social Choice Rules

The regime chosen by majority party legislators depends on the social choice procedure used within the party caucus. With a couple of intuitive jumps it is possible to give a rough characterization of regime selection under majority rule or unanimity rule before proceeding to a more detailed analysis.

Centrist legislators prefer a carrot regime to either other regime. Under a carrot regime they will be compensated for their costs of voting for the party's platform, whereas under the stick regime they will be uncompensated. The relative preferences of centrist legislators over the stick regime and non-delegation are a bit more complex. If a legislator has a low enough cost of voting for the party's platform, then he may prefer a stick regime to non-delegation as he may be willing to incur the cost of voting for the party's platform and gain the partisan utility B of passing the party's platform.

Stalwart legislators generally prefer a stick regime to either other regime. Under a stick regime there will be no cost to stalwart legislators, whereas under a carrot regime they will have to compensate centrist legislators for voting for the party's platform. Depending on the mechanism selected in the carrot regime stalwart legislators may or may not prefer this regime to non-delegation.

Under majority rule, the regime chosen depends on whether stalwarts or centrists make up a majority of the party. A carrot regime would be chosen whenever centrists make up a majority of the party, $n > \frac{m}{2}$, and a stick regime would be chosen whenever stalwarts make up a majority of the party $n < \frac{m}{2}$. Assuming majority rule, however, ignores the very real possibility that the regime chosen by the majority group of the party may cause members of the minority group to bolt the party. Paradoxically, majority rule within the party caucus pre-supposes that their is enough party influence to keep the caucus together in a floor vote

on how much influence to give party leaders.

This problem might be circumvented by unanimity rule. As will be seen in the next section, a carrot regime is sometimes unanimously preferred to non-delegation. It is even possible (though unlikely) for a stick regime to be unanimously preferred to non-delegation. However, neither a carrot nor a stick regime is unanimously preferred to the other.

Rather than selecting a particular social choice rule, in order to derive comparative statics I instead assume that the social choice procedure has the following properties:

- A carrot regime is more likely to be chosen as stalwart legislators' expected utilities increase under that regime.

- A stick regime is more likely to be chosen as the number of centrist legislators that prefer that regime to non-delegation increases.[11]

Given a social choice rule that satisfies these properties, legislators will design the mechanism to be implemented in the carrot regime to increase its appeal to stalwart legislators and will design the mechanism to be implemented in the stick regime to increase that regime's appeal to centrist legislators. To achieve these goals, mechanisms should have the following properties:

Minimize Costs: The mechanism should force only centrist legislators with the lowest costs to vote for the party's platform. This will reduce the compensation that will need to be paid by stalwart legislators in the carrot regime, and reduce the expected costs borne by centrist legislators in the stick regime. More formally, a mechanism minimizes costs if and only if the centrist legislators with the lowest costs (revealed ex-post) of voting for the party's platform have to do so.[12]

[11] If neither a carrot or stick regime is adopted the result is non-delegation where party leaders are given no ability to reward or punish votes of party legislators.

[12] Note that this means minimal winning coalitions will be preferred to coalitions of other sizes due to the assumption that there is only a single vote-buyer.

There is another reason legislators will prefer a mechanism that minimizes costs. When a legislator votes against his district's interests he suffers an electoral cost. While the legislator may receive some

Ex-Post Efficiency: The mechanism should pass the party's platform only when it is socially efficient (among party legislators) to do so. If the mechanism passes the platform when it is not ex-post efficient, the compensation granted to or costs borne by legislators will exceed the partisan utility of legislators. To formalize this criteria, label centrist legislators in order of their position-taking costs (which become common knowledge after the game is finished). That is, $i \in \{1, 2, 3, \ldots, n\}$ where $v_1^c \leq v_2^c \leq v_3^c \leq \cdots \leq v_n^c$. Then, a mechanism is ex-post efficient if the party's platform passes if and only if

$$mB - \sum_{j=1}^{k} v_j^c > 0. \tag{1}$$

Truthful Revelation: In any mechanism legislators will be asked to submit bids b that indicate their cost of voting for the party's platform. In a truthful revelation mechanism it is a dominant strategy for centrist legislators to truthfully report their position-taking costs. Formally, in a dominant strategy equilibrium, for each centrist legislator i, $b_i = v_i^c$.

A caveat is in order before designing the optimal mechanism within each regime. Even if a centrist legislator would prefer a stick regime to non delegation, he would not be likely to vote for a stick regime. By voting for the stick regime, he would be revealing that his position-taking cost is low. Party leaders, wishing to minimize costs are more likely to threaten a centrist legislator that expressed his preference for a stick regime over non-delegation. This would lower the expected benefit of the stick regime for the centrist legislator in question, which may cause him to change his vote. Thus, I assume throughout that party leaders are not allowed to use any information revealed by the vote on the regime in designing the carrot

compensation for his vote, this compensation may be partly non-electoral, such as the promise of a lobbyist job after retirement from the legislature. A reduction in a given legislator's probability of re-election also reduces the probability that other party legislators will be in the majority in the following Congress. All else equal legislators prefer to maximize the chance that they are in the majority again because only the majority party can enact its platform.

or stick mechanism.[13]

3 Carrots

The use of carrots—rewards conditional on a legislator's vote on the party's platform—has been well studied when there is complete information about legislator preferences. Party leaders will simply pay those legislators with the lowest costs of voting for the platform enough to make them exactly indifferent between voting for and against the platform. In this scenario, the platform passes if and only if (1) holds, so this mechanism is efficient. Truthful revelation is not an issue here and costs are minimized so this mechanism satisfies all three desirable properties from the previous subsection.

When legislator preferences are private information, the optimal use of carrots has received less attention. Fortunately, the problem resembles several in auction theory.

I focus on a uniform auction. In a uniform auction, centrist legislators submit bids b_i, which, in equilibrium are their position-taking costs of voting for the platform $b_i = v_i^c$. Party leaders pay the legislators who submit the k lowest bids the $k + 1^{st}$ lowest bid.

Proposition 1 *The uniform auction is a dominant strategy mechanism that minimizes costs.*

Proof. Since each legislator can only sell a single vote, the uniform auction is the same as the Vickery (1961) auction. As legislator costs are drawn independently, it is a weakly dominant strategy for a legislator to bid his value in the Vickery auction.

Consider the strategy of centrist legislator i, when all other centrist legislators bid b_{-i}. Order the bids of other centrist legislators such that $b_1 \leq b_2 \leq \cdots \leq b_{n-1}$. If legislator i

[13] A secret ballot may not be enough, as which regime wins the vote may reveal useful information about the number of centrist legislators who find a stick regime palatable. Alternatively, I could assume that legislators vote before they discover their costs of voting for the party's platform (although they know whether they will be a centrist or an stalwart). This produces similar results although more distributions of parameters would need to be defined, and results would be expressed in terms of expectations over expectations.

bids his position-taking cost v_i^c, then he receives a payoff of $B + b_k - v_i^c$ if $v_i^c < b_k$ and B otherwise.

Consider the case where $v_i^c \geq b_k$. If legislator i bids any amount greater than b_k, his payoff does not change. If legislator i bids some amount less than b_k, he receives $B + b_k - v_i^c$, which is less than B.

If, instead, $v_i^c < b_k$, then bidding an amount greater than b_k decreases the legislator's payoff to B. Any bid less than b_k does not change the payoff. Since b_{-i} is arbitrary, it is a weakly dominant strategy for a legislator to bid his position-taking cost. If all centrist legislators bid their position-taking costs, then the uniform auction minimizes costs—only those legislators with the lowest costs are paid to vote for the party's platform. ■

Denoting the $k+1^{st}$ lowest bid (which, in equilibrium is the $k+1^{st}$ lowest position-taking cost) by $b_{k+1} = v_{k+1}^c$, the uniform auction pays k centrist legislators v_{k+1}^c for total payments of kv_{k+1}^c. However, the platform will pass even when (1) does not hold, violating ex-post efficiency. In that case stalwart legislators will pay more than their partisan utility B.

With two modifications the uniform auction will ensure that stalwart legislators never pay more to pass the platform then it is worth to them. This increases the probability that a carrot regime will be adopted. These modifications, discussed below, are to establish a *reserve* r and to tax all centrist legislators an amount B when party leaders are able to buy enough votes to pass the platform.

A reserve r caps the maximum amount paid for votes. If $b_k = v_k^c \leq r < v_{k+1}^c = b_{k+1}$, then in equilibrium centrist legislators with position-taking costs $v_i^c \leq v_k^c$ will be paid r. If $v_k^c > r$ then all centrist legislators will be paid zero, and the platform will fail. If $v_{k+1}^c < r$ then the uniform auction works exactly as before.

The reserve should be set such that when the platform passes the utility of stalwart legislators will be at least as great as the value of having non-delegation (namely zero).[14]

[14]This is essentially the same as giving stalwart legislators an ex-post veto.

That is, $B \geq \frac{kr^*}{m}$, or

$$r^* = r^*(m, B, k) = \frac{mB}{k}. \tag{2}$$

The next proposition shows that these modifications do not affect the desirable properties of the uniform auction.

Proposition 2 *A uniform auction with reserve r^* that taxes all centrist legislators an amount B if and only if $v_k^c \leq r^*$ is a dominant strategy mechanism that minimizes costs.*

Proof. Consider the strategy of centrist legislator i, when all other centrist legislators bid b_{-i}. Order the bids of other centrist legislators such that $b_1 \leq b_2 \leq \cdots \leq b_{n-1}$. If $b_k < r^*$ then the logic is the same as in the proof to Proposition 1, but with all payoffs decreased by an amount B.

If $b_{k-1} > r^*$ then any bid will result in the same payoff for legislator i, namely 0. If $b_{k-1} < r^* < b_k$ then when legislator i bids his position-taking cost v_i^c he will receive $r^* - v_i^c$ if $v_i^c < r^*$ and 0 otherwise. If $v_i^c < r^*$ if i bids any amount less then r^* his payoff will be the same. If i bids an amount greater than r^* his payoff will decrease to zero.

If instead $v_i^c \geq r^*$ then any bid greater than r^* will not change the utility of legislator i, whereas a bid of less than r^* would decrease i's payoff to $r^* - v_i^c < 0$. Since b_{-i} is arbitrary, it is a weakly dominant strategy for legislator i to bid his true cost. If all centrist legislators bid their true costs the mechanism minimizes costs—only those legislators with the lowest costs will be paid to vote for the party's platform, and only when $b_k = v_k \leq r^*$. ■

While the uniform auction satisfies two of the properties in Section 2.2, it may seem counter-intuitive to pay all centrist legislators who vote for the party's platform the same amount. Could party leaders reduce the amount spent by paying each legislator that votes for the party's platform his bid?

The short answer is no. This alternative mechanism, called a discriminatory auction, results in the same expected payoffs to legislators, and thus the same expected payments

from party leaders. Showing this is beyond the scope of this paper, but some intuition is possible. The equilibrium bidding strategies in the discriminatory auction are different from those in the uniform auction. Specifically, centrist legislators will adopt a bidding strategies to ensure the same expected profit from the sale of their vote as they would have had in the uniform auction. This implies that the discriminatory auction is not a truthful revelation mechanism.

Moreover, both the discriminatory and uniform auction have the same expected costs to party leaders as auctioning off votes one-by-one using either first-price (pay your bid) or second-price (pay the first loser's bid) auctions. In fact, *any* mechanism that minimizes costs produces the same costs for party leaders.[15] All of these mechanisms minimize costs, but the uniform auction is a dominant strategy mechanism, and thus, strategically simple.

There are a few things to note here. First, this mechanism is not ex-post efficient. In fact, there is no ex-post efficient mechanism that does not give stalwarts negative utility for some parameter values.[16] This occurs because centrist legislators need to be compensated for truthfully revealing their position-taking costs.[17]

Second, stalwart legislators are generally charged less than their partisan utility B by the mechanism. However, all centrist legislators must be charged exactly B (minus any compensation for voting for the party's platform).[18] The modified uniform auction is a dominant strategy mechanism in part because the payment to a centrist legislator does not depend on that legislator's bid (conditional on the bid being one of the k lowest bids). Thus, while it is possible to charge stalwarts an amount that depends on the amount spent in the

[15]This is called the revenue equivalence principle. While it is easy to understand why different mechanisms would have different equilibrium betting strategies, it is quite difficult to explain conceptually why each mechanism will result in the exact same costs. It is easy to show this mathematically, however doing so would require defining a large amount of new notation, and would be essentially repeating the results in Krishna and Perry (1998).

[16]The unmodified uniform auction is ex-post efficient, but gives stalwart legislators negative utility whenever $v^c_{k+1} > r^*$.

[17]This compensation is often referred to as an information rent.

[18]If centrist legislators were charged a fixed amount less than B then charging them more would result in greater expected utility for stalwart legislators, making stalwarts more likely to support a carrot regime.

auction, it is not possible to do this for centrist legislators.[19]

Finally, when $v^c_{k+1} < \frac{nB}{k}$ the modified uniform auction gives stalwart legislators utility greater than their partisan utility B. Thus, for certain distributions of centrist legislator costs $F(v^c)$, stalwart legislator's expected utility will be larger than B under a carrot regime. As will be seen in Section 5, under such distributions stalwarts will strictly prefer a carrot regime to a stick regime.

4 Sticks

While vote buying models are ubiquitous in political science, it is less common to consider the use of contingent punishments. To develop intuition, this section first analyzes a model where party leaders 1) are completely informed about legislators' position-taking utilities and 2) are not answerable to the rank and file. The final subsection returns to the dual-utility model where party leaders are elected by party legislators and have only incomplete information.

4.1 What are Sticks?

A stick is a threat. A party leader informs a legislator that if he takes a certain action the party will inflict some level of harm on him.

In a real world legislature, what can a party leader threaten? A simple example is a threat to remove a legislator from a prestigious committee (Baker, 1985). For more senior legislators, a party leader could threaten the loss of a committee or subcommittee chair. A final threat that could be applied to all legislators is to support (and sometimes even recruit) a candidate to run against the legislator in his next primary. For example, in 2002 the Republican party supported John Sununu's successful bid to unseat incumbent Republican

[19]This doesn't matter from the perspective of getting the carrot regime adopted, centrist legislators will still strictly prefer it to a stick regime.

Senator Bob Smith.[20]

Exercise of these threats is costly to the party. In the case of removing a committee chair this cost may be small: the party leader will have to bear the erstwhile chairman's wrath, and the committee may lose valuable human capital. In the case of running a primary challenger these costs can be substantial. Throughout it is assumed it costs party leaders an amount x to inflict a cost of x on a legislator.

It is important to note that a stick is not a negative carrot. That is, using a stick against a legislator does not increase the resources at the disposal of party leaders. Sticks are inherently destructive: their use reduces the utility of both the legislators that pay for the stick, and the legislator on whom the stick is used. By contrast a carrot is a transfer that increases the utility of a legislator while reducing the utility of the legislators that pay for the carrot. To put this another way: in the carrot regime legislators are playing a zero-sum game, while in the stick regime legislators are playing a negative-sum game.

4.2 Complete Information

To develop an intuition for how mechanisms in the stick regime work and to facilitate contrasts with vote buying models, this subsection analyzes a modified version of the dual-utility model. In this variant, postion taking utilities are common knowledge to all legislators and party leaders are not elected by legislators. Thus party leaders will pass the party's platform whenever they can, regardless of whether it is ex-post efficient to do so. Further, for this subsection only, party leaders will be assumed to have a budget D to pay for punishments. This prevents party leaders from issuing threats they will not be able to carry out due to insufficient resources.[21]

[20]Whatever benefit is threatened must, with some known probability, accrue to the legislator if the party leader did nothing. Thus, to the extent that legislators might want to make threats available to party leaders they may want to take certain benefits that are granted as rewards and grant them as a matter of course. For example, committee chairs could be awarded at the whim of party leaders as a reward. If committee chairs were instead allocated by a seniority system, this would create an expectation of a benefit that could be threatened by party leaders.

[21]In the carrot regime this budget balancing condition is imposed by the reserve price which guarantees that the total amount of payments needed will not be greater than the benefit of passing the party's platform.

A stick can induce compliance from a centrist legislator at zero cost to party leaders.[22] Consider centrist legislator i with position-taking cost v_i^c. If party leaders threaten to punish legislator i an amount greater than v_i^c *if legislator i votes against the party's platform*, legislator i will vote for the party's platform. As legislator i has complied with the party leader's demand, no punishment is issued. As party leaders can change a vote without expending any resources, the full budget D may be used to threaten multiple legislators. Thus, any centrist legislator with position-taking costs $v_i^c < D$ can be threatened into voting for the party's platform simultaneously.

This outcome seems quite fragile. If there were two legislators i and j such that $v_i^c, v_j^c < D$ but $v_i^c + v_j^c > D$, couldn't i and j agree to vote for the platform, and in expectation profit? More succinctly, is the equilibrium in the previous paragraph *coalition-proof*?

Definition 1 *An equilibrium is said to be* **coalition-proof** *if no group can make its legislators at least as well off by playing a strategy other than that prescribed by the equilibrium.*

Provided that party leaders have sufficient resources to threaten the pivotal legislator of the chamber, it is always possible for party leaders to create a coalition-proof equilibria.[23]

Proposition 3 *If $v_k^c < D$ then there exists a coalition-proof Nash equilibrium where the party's platform passes at zero cost to party leaders.*

Proof. In a coalition-proof Nash equilibrium, party leaders create an ordered list of the centrist legislators they are threatening. The party leaders declare to each legislator on the list, "If you are the first legislator on this list that votes against the party's platform, you will suffer a punishment of size D." It is now no longer rational for the first legislator on this list to vote against the platform, so he does not. The second legislator on the list knows that he will bear a punishment of size D if he votes against the party's platform and thus will vote for the platform, and so on. ■

[22] This would not be the case if there was a cost associated with simply making a threat.
[23] It is also necessary that centrist legislators cannot transfer utility to each other.

Intuitively, centrist legislators are trying to solve a collective action problem by banding together. As detailed in the proof above, party leaders simply re-create the collective action problem. Although legislators are threatened simultaneously, by making an ordered lists of who will be punished it is *as if* centrist legislators are being threatened, and voting for the party's platform, sequentially

In the stick regime with complete information where party leaders are not accountable to rank-and-file legislators, super-majority coalitions are generally feasible. In addition, an observer who can see transfers or punishments but not threats will not observe party leaders doing anything. In other words, it will appear as though the party leaders have no influence over legislative outcomes.

Corollary 1 *Find j such that $v_j^c \leq D \leq v_{j+1}^c$. Then if $j > k$ there exists a coalition-proof Nash equilibrium where the party's platform passes with a supermajoritarian coalition of size $(m-n)+j > \frac{|\mathcal{L}|+1}{2}$ at zero cost to party leaders. Further, this equilibrium will involve no transfers or punishments.*

Proof. As $v_j^c < D$ party leaders can threaten all legislators i where $i \leq j$ using the strategy in the proof of Proposition 3. In this equilibrium all centrist legislators $i \leq j$ will vote for the party's platform. This yields a coalition size of $(m-n)+j > (m-n)+k > \frac{|\mathcal{L}|+1}{2}$ by definition of k. In any equilibrium of this form each threatened legislator will vote for the party's platform and thus no punishments or transfers are used. ■

Clearly, in a stick regime party leaders would have almost unlimited power to pass the party's platform if they: 1) had complete information about the costs of legislators in voting for the platform 2) were not bound by rules selected by rank and file legislators. The next section considers what happens when these two constraints are added, returning to the dual-utility model.

4.3 Incomplete Information

Imagine party leaders ran an ascending payment discriminatory auction to determine which centrist legislators should be threatened into voting for the party's platform. In this mechanism, party leaders slowly raise a price, starting at zero, that is seen by all legislators. Legislators drop out to indicate their bid b_i, and are threatened with a punishment of that size. Once k centrist legislators drop out, the auction ends.

In this mechanism, every centrist legislator should drop out immediately, thereby indicating that they would vote for the platform if threatened with a punishment of size zero. Facing a threat of punishment of size zero, each legislator will, of course, vote against the party's platform and suffer no consequences. While this example explores a particular mechanism, the inability to determine which legislators to threaten such that costs are minimized is endemic to the stick regime.

Proposition 4 *There does not exist any mechanism in the stick regime that minimizes costs.*

Proof. Suppose a stick mechanism that minimizes costs exists to show a contradiction. Using the revelation principle (Myerson, 1981) this is equivalent to a truthful revelation mechanism.[24] In the equilibrium of this mechanism, each centrist legislator submits his position-taking cost. As this mechanism is assumed to minimize costs, the k legislators with the lowest position-taking costs are threatened into voting for the bill.

Consider legislator k, with the k^{th} lowest position-taking cost. Does this legislator want to truthfully reveal his position-taking utility, given that all other legislators truthfully reveal their position-taking costs? If k does, he will be forced to vote for the platform and incur a cost of v_k^c. If he reports the maximum possible cost $\overline{v}$ instead, he will not be threatened, and incurs no costs. Clearly he prefers the later strategy, falsifying the existence of a mechanism that minimizes costs. ■

[24] It is straightforward to verify the revelation principle holds in this non-standard setting.

A stick mechanism that minimizes costs does not exist because the centrist legislators that are threatened into voting for the platform are worse off then those who are allowed to vote against the platform.[25] In the absence of a mechanism that minimizes costs as defined in Section 2.3, the cheapest mechanism for centrist legislators is for party leaders is to randomly select $k+1$ centrist legislators and threaten to punish them an amount at least $\overline{v}$ if they vote against the party's platform.[26] Assumption 1 guarantees that $\overline{v} < mB$ so these threats are credible and can be implemented in a coalition-proof equilibrium, as in the previous section.

Having designed the mechanisms legislators would select in either regime, I turn to the question of how legislators would fare under each regime.

5 Comparing Regimes

Stalwarts and centrists have different preferences over regimes. Using the social choice principles described in Section 2.3 the likelihood each regime will be chosen depends on how much more (in an expected utility sense) stalwarts like a stick regime than a carrot regime, and how many centrist legislators prefer a stick regime to non-delegation. This section examines how these quantities change with the parameters of the model.

The parameters of the model are m the size of the party, n the number of centrist legislators, k the number of centrists whose vote the party needs to pass its platform, B the partisan utility of all party legislators, and $F(v_i^c)$ the distribution of the position-taking costs of centrist legislators. The parameters m, n and k will not change independently. For these parameters there are three scenarios to consider: increasing the size of the party by adding either an stalwart or a centrist legislator and a stalwart legislator becoming a centrist legislator.

The first scenario occurs when the party wins an additional seat in a district that prefers

[25] While it might still be possible to determine which centrist legislators have the lowest costs of voting for the platform, to do so it would be necessary for the other legislators to burn some amount of utility (in this case electability) to prove their costs are high. This would not minimize costs as defined in Section 2.3.

[26] Party leaders cannot threaten only k legislators since if a single centrist legislator defected, the benefit mB would not be realized by party legislators, and there would be nothing to fund the required punishment.

the party's platform. The second scenario occurs when the party wins a centrist district formerly held by the other party. The final scenario is that a legislative district that had formerly supported the party's platform no longer does due to shifts in the platform or shifts in district characteristics.

5.1 Stalwart Legislators

Under the properties of the social choice rule outlined in Section 2.3 the carrot regime is more likely to be adopted as the expected utility of stalwart legislators increases. To derive the expected utility of a stalwart legislator under the carrot regime note that the party's platform will only pass if and only if:

$$v_k^c \leq r^* = \frac{mB}{k} \tag{3}$$

The surplus $S(v_{k+1}^c; k, m, n, B)$ of a stalwart legislator in the carrot regime, for a given value of $v_{k+1}^c \leq r^*$ is

$$S(v_{k+1}^c; k, m, n, B) = B - \frac{kv_{k+1}^c - nB}{m - n}. \tag{4}$$

Thus, the expected utility of a stalwart legislator under the carrot regime is

$$\int_0^{r^*} \left(B - \frac{kv_{k+1}^c - nB}{m - n} \right) dF_{k+1}^n(v_{k+1}^c) + \left(B - \frac{kr^* - nB}{m - n} \right) P(v_k^c < r^* < v_{k+1}^c) \tag{5}$$

where $F_{k+1}^n(v)$ as the c.d.f. of the $k+1$ order statistic ($k+1$ lowest value from n draws).[27] That is

[27]Note that in auction theory the $k+1$ order statistic is the $k+1$ *highest* draw. Because auction theorists decided to upend the traditional mathematical notation to suit themselves, I feel no guilt in ignoring them and using traditional mathematical notation. See Krishna (2002, Appendix C).

$$
\begin{aligned}
Pr\left(v_{k+1}^{c} \leq v^{c}\right) &= F_{k+1}^{n}\left(v^{c}\right) \quad \text{where} \\
F_{k}^{n}\left(v^{c}\right) &= \sum_{j=0}^{n-k} \left(\begin{array}{c} n \\ j \end{array} \right) F(v^{c})^{n-j}(1-F(v^{c}))^{j} \qquad (6)
\end{aligned}
$$

where $F(v^c)^n = (F(v^c))^n$. Note that the second term in (5) is zero by the definition of r^*.

Outside observers will generally not be able to distinguish between party leaders that are not empowered by legislators and those that do not have sufficient power (or resources) to change legislative outcomes. Thus, it is also necessary to consider the probability that the party's platform passes under the carrot regime. This probability is $P(v_k^c < r^*) = F_k^n(r^*)$ and, except where noted, increases when the expected utility of stalwart legislators increases.

Changes in the parameters of the model change the expected utility of stalwart legislators through three channels. They can change the value of the surplus $S(v_{k+1}^c)$ for a given value of v_{k+1}^c. A change in parameters may also change the reserve r^*, and thus the limits of integration in (5). Finally, a change in the parameters of the model may change the c.d.f. of the order statistic used in the integral in (5) by changing either the number of draws (k) or the size of the pool from which the order statistic is drawn (n). The following lemmas detail how these changes will change the expected utility of stalwart legislators.

Lemma 1 *A change in parameters that simultaneously increases $S(v^c; k, m, n, B)$ for all values of $v^c \in [0, r^*]$, and increases r^* increases the expected utility of stalwart legislators under the carrot regime.*

Proof. Consider a change in parameters from p to p' which increases $r^*(p)$ to $r^*(p) + \Delta r^*(p')$ and $S(p)$ to $S(p) + \Delta S(p')$ for all $v^c \in [0, r^*(p) + \Delta r^*(p')]$. $r^*(p) + \Delta r^*(p')$ is defined such that $S(p) + \Delta S(p') > 0$ for all $v^c \in [0, r^*(p) + \Delta r^*(p')]$ thus,

$$
\int_0^{r^*(p)+\Delta r^*(p')} [S(p) + \Delta S(p')]\, dF_{k+1}^n(v^c) > \int_0^{r^*(p)} S(p) dF_{k+1}^n(v^c) \quad + \int_{r^*(p)}^{\Delta r^*(p')} S(p) dF_{k+1}^n(v^c) > \int_0^{r^*(p)} S(p) dF_{k+1}^n(v^c).
$$

■

To understand the effects of a change in the c.d.f. of the order statistic in the integral in (5), it is useful to use the concept of *first order stochastic dominance* (FOSD). When a distribution $G(\cdot)$ FOSD $F(\cdot)$, it is analogous to saying that $G(\cdot)$ is greater than $F(\cdot)$. For example, if x is distributed according $F(x)$, and y is distributed according to $G(y)$ then the expectation of y is no less than the expectation of x, $E(y) \geq E(x)$.

Definition 2 *A distribution $G(v^c)$* **first order stochastic dominates** *(FOSD) a distribution $F(v^c)$ if for all $v \in [0, \overline{v}]$*

$$\int_0^{v^c} g(x)dx = G(v^c) \leq F(v^c) = \int_0^{v^c} f(x)dx \tag{7}$$

As the definition suggests, a distribution FOSD another distribution when more of the mass of its probability distribution function is bunched towards the upper limit of the support of the distribution.

Lemma 2 *If a distribution $G(v^c)$ first order stochastic dominates a distribution $F(v^c)$, then stalwart legislator expected utility at least as great under $F(v^c)$ as under $G(v^c)$.*

Proof. Since the second term in (5) is zero, the difference in utilities under $F(v^c)$ and $G(v^c)$ is given by:

$$\int_0^{r^*} \left(B - \frac{kv^c - nB}{m-n} \right) (f(v^c) - g(v^c))dv^c$$

Integration by parts yields:

$$\frac{mB}{m-n}(F(r^*) - G(r^*)) + \frac{k}{m-n}\int_0^{r^*} (F(v^c) - G(v^c))dv^c \geq 0$$

where both terms are non-negative since $F(v^c) \geq G(v^c)$. ■

These lemmas will allow us to examine the relevant changes in the parameters of the model. In all the cases below, the probability that the party's platform passes under the

carrot regime increases when the expected utility of stalwart legislators increases, and the probability decreases when the expected utility of stalwart legislators decreases, except where explicitly noted.

Adding a Stalwart Legislator increases m by one and decreases k by one. This increases the surplus for a given value of v^c, $S(v^c, k-1, m+1, n, B) > S(v^c, k, m, n, B)$, as there are fewer centrist legislators that need to be bought, and more legislators to bear the costs of buying those legislators. The reserve, $r^*(k-1, m+1, B) > r^*(k, m, B)$ increases for the same reasons. Finally, the distribution of the k^{th} order statistic is first order stochastic dominated by the distribution of the $k+1^{st}$ order statistic. This is shown formally in the appendix, but intuitively, the probability that the $k+1^{st}$ lowest draw is less than some number must always be less than the probability that the k^{th} lowest draw is less than that number. Taking these results together with Lemmas 1 and 2 implies the expected utility of stalwart legislators under the carrot regime increases when a stalwart legislator is added.

Adding a Centrist Legislator: increases both n and m by one. This increases both the surplus $S(v^c, k, m+1, n+1, B) > S(v^c, k, m, n, B)$ and the reserve $r^*(k, m+1, B) > r^*(k, m, B)$, since the number of legislators who can bear the costs of buying the k centrist legislators has increased. As shown in the appendix, $F^n_{k+1}(v^c)$ FOSD $F^{n+1}_{k+1}(v^c)$. Intuitively, when adding another draw ($n+1$ rather than n) this decreases the expected value of the $k+1^{st}$ lowest draw. Taking these results with Lemmas 1 and 2 implies the expected utility of stalwart legislators under the carrot regime increases when an centrist legislator is added.

A Stalwart Legislator Shifts to a Centrist Legislator: increases n and k by one. This has several effects that push in opposite ways. On the one hand, the reserve r^* decreases, and the new distribution $F^{n+1}_{k+2}(v^c_{k+2})$ FOSD $F^n_{k+1}(v^c_{k+1})$ (as shown in the appendix). Both of these effects will decrease the expected utility of stalwart legislators.

On the other hand the surplus for a given value of v^c_{k+1} may either increase or decrease. The net effect of these changes will generally be to decrease the expected utility of stalwart legislators as surplus will decrease for higher values of v^c_{k+1}, which are now more likely to occur.

Additionally, it is straightforward to show that the probability the party's platform passes under the carrot regime will decrease:

$$F^{n+1}_{k+1}\left(\frac{mB}{k+1}\right) \leq F^{n+1}_{k+1}\left(\frac{mB}{k}\right) \leq F^{n}_{k}\left(\frac{mB}{k}\right)$$

where the first inequality is strict when $\frac{mB}{k+1} < \overline{v}$ and the second inequality is strict when $\frac{mB}{k} < \overline{v}$. When a stalwart legislator becomes a centrist legislator it becomes no more likely that party leaders will use rewards to pass the party's platform in the carrot regime. Intuitively, there is now one more legislator that needs to be bought in order for the party's platform to pass. This increases the demand for centrist votes and increases the average amount the party will need to pay in order to pass its platform.

Partisan Utility Increases: This is an increase in B. Both the surplus $S(v^c_{k+1})$ and the reserve r^* increase as there will be more resources left over when after the same centrist legislators have been bought. The distribution $F^n_{k+1}(v^c_{k+1})$ does not change, so overall, an increase in B increases the expected utility of stalwart legislators under the carrot regime.

An Increase in the Distribution of Centrist Legislator Costs: Suppose centrist legislators' position-taking costs are now distributed according to a distribution $G(v^c)$ that FOSD $F(v^c)$. If the probability that v^c_i is larger increases (as it does under $G(v^c)$ compared to $F(v^c)$) then the probability that v^c_{k+1} is larger also increases.[28] Combining this fact with Lemma 2, an increase in the distribution of centrist legislator costs will decrease the expected utility of stalwart legislators under the carrot regime. In-

[28]Formally, the appendix shows that if $G(v)$ FOSD $F(v)$ then $G^n_{k+1}(v)$ FOSD $F^n_{k+1}(v)$

tuitively, the rewards that will need to be payed are in expectation larger, decreasing the expected utility of stalwart legislators.

Using the first social choice principle described in Section 2.3, on the one hand it is more likely that a carrot regime will be adopted, and will pass the party's platform when the size of the party increases and the benefit to party members of passing the platform increases. On the other hand, the probability that the carrot regime will be adopted and will pass the party's platform decreases when the proportion of the party that is centrist increases, or when the position-taking costs of centrist legislators stochastically increase.

5.2 Centrist Legislators

Under the second property of the social choice rule in Section 2.3 the stick regime is more likely to be adopted as the expected number of centrist legislators that prefer it to non-delegation increases. To derive the expected number of centrist legislators that prefer the stick regime to non-delegation, note that a stick regime results in a $\frac{k+1}{n}$ chance that a centrist legislator has to pay his position-taking costs without compensation. All legislators have a utility of zero when there is non-delegation, so a centrist legislator i ex-ante prefers a stick regime to non-delegation if:

$$\frac{k+1}{n} v_i^c \leq B$$

Of course, if a given centrist legislator is randomly chosen to vote for the party's platform then ex-post he will wish that non-delegation had been selected.

The probability that a given centrist legislator prefers a stick regime to non-delegation is $F\left(\frac{nB}{k+1}\right)$. The number of centrist legislators that prefer a stick regime to non-delegation by z, the probability that there are exactly j such centrist legislators is given by:

$$Pr\left(z=j\right) = \left(\begin{array}{c} n \\ j \end{array} \right) F\left(x\right)^j \left(1-F\left(x\right)\right)^{n-j} \tag{8}$$

where $x = \frac{nB}{k+1}$. The expected number of centrist legislators $E\left(z|m,k,n,B\right)$ that prefer a

stick regime to non-delegation can be calculated using c.d.f.'s of order statistics as defined in (6):

$$\begin{aligned} E(z|k,m,n,B) &= \sum_{j=0}^{n}(n-j)\left(\begin{array}{c} n \\ n-j \end{array}\right) F(x)^{n-j}\left(1-F(x)\right)^{j} \\ &= nF_n^n(x)+(n-1)\left(F_{n-1}^n(x)-F_n^n(x)\right)+\ldots \\ &\qquad \cdots+(n-(n-1))\left(F_2^n(x)-F_{n-1}^n(x)\right) \\ &= \sum_{j=1}^{n}F_j^n(x)=\sum_{j=1}^{n}F_j^n\left(\frac{nB}{k+1}\right) \end{aligned} \tag{9}$$

Note that $E(z|k,m,n,B)$ is a sum of increasing functions and is thus an increasing function. This particularly convenient representation allows the computation of the same comparative statics as in the previous subsection.

Adding a Stalwart Legislator increases m by one, and decreases k by one:

$$\begin{aligned} F_j^n\left(\frac{nB}{k}\right) &\geq F_j^n\left(\frac{nB}{k+1}\right) \Rightarrow \\ E(z|k-1,m+1,n,B)=\sum_{j=1}^{n}F_j^n\left(\frac{nB}{k}\right) &\geq \sum_{j=1}^{n}F_j^n\left(\frac{nB}{k+1}\right)=E(z|k,m,n,B) \end{aligned}$$

where the inequality is strict when $\frac{nB}{k+1}\leq\overline{v}$. Adding a stalwart legislator does not affect the potential pool of centrist legislators to be threatened, but does reduce the number that need to be threatened. The later effect reduces the probability that any one centrist legislator is threatened, which increases the expected value of a stick regime to centrist legislators. This increases the expected number of centrist legislators which would prefer a stick regime over non-delegation.

Adding a Centrist Legislator increases both n and m by one:

$$\begin{aligned} E(z|k, m+1, n+1, B) &= \sum_{j=1}^{n+1} F_j^{n+1}\left(\frac{(n+1)B}{k+1}\right) > \sum_{j=1}^{n} F_j^{n+1}\left(\frac{(n+1)B}{k+1}\right) \geq \\ &\geq \sum_{j=1}^{n} F_j^{n+1}\left(\frac{nB}{k+1}\right) \geq \sum_{j=1}^{n} F_j^{n}\left(\frac{nB}{k+1}\right) = E(z|k, m, n, B) \end{aligned}$$

where the weak inequalities are strict when $\frac{nB}{k+1} \leq \overline{v}$. Adding a centrist legislator increases the total pool of centrist legislators who can be threatened, lowering the probability that any given legislator might be threatened. This raises the expected value of a stick regime for a centrist legislator, which increases the expected number of centrist legislators who would prefer a stick regime over non-delegation.

A Stalwart Legislator Shifts to a Centrist Legislator: increases n and k by one. This has two effects. The first effect raises the probability that a centrist legislator is threatened:

$$F_j^{n+1}\left(\frac{(n+1)B}{k+2}\right) \leq F_j^{n}\left(\frac{(n+1)B}{k+2}\right) \leq F_j^{n}\left(\frac{nB}{k+1}\right)$$

where the first inequality is strict when $\frac{(n+1)B}{k+2} \leq \overline{v}$, and the second is strict when $\frac{nB}{k+1} \leq \overline{v}$. In isolation this would raise the expected number of centrist legislators that would support a stick regime. Another effect pushes in the other direction, however. There are now more centrist legislators so even if a smaller proportion of them prefer a stick regime this can still lead to a larger expected number of centrist legislators that prefer a stick regime to non-delegation:

$$\sum_{j=1}^{n+1} F_j^{n+1}\left(\frac{(n+1)B}{k+2}\right) > \sum_{j=1}^{n} F_j^{n+1}\left(\frac{(n+1)B}{k+2}\right)$$

Thus, when an stalwart legislator shifts to a centrist legislator, the expected proportion of centrist legislators preferring a stick regime to non-delegation increases. However, the total number of centrist legislators that prefer a stick regime to non-delegation may

either increase of decrease.

Partisan Utility Increases: An increase in B increases the number of centrist legislators that prefer a stick regime to non-delegation as $F_j^n(\cdot)$ is an increasing function, as shown in the Appendix.

An Increase in the Distribution of Centrist Legislator Costs: A change in the distribution $F(\cdot)$ of centrist legislator position-taking costs to a distribution $G(\cdot)$ that first order stochastic dominates it decreases the expected number of centrist legislators that prefer a stick regime to non-delegation (since $\forall x,\ G_j^n(x) \leq F_j^n(x)$). This occurs because each centrist legislator now has a higher probability of obtaining a value v_i^c that makes the expected value of a stick regime negative.

Using the second social choice principle described in Section 2.3 it is more likely that a stick regime will be adopted when the size of the party increases and the benefit to party members of passing the platform increases. On the other hand, the probability that the stick regime will be adopted decreases when the proportion of the party that is centrist increases, or when the position-taking costs of centrist legislators stochastically increase.

When interpreting the comparative statics above, one must keep in mind that with one exception the comparative statics in both of these subsections move in the same direction as summarized in Table 2. Thus, almost anything that makes a carrot regime more likely (using the social choice principles in Section 2.3) will also make a stick regime more likely. So while it is possible to say that increasing the size of the party or the benefit to legislators of passing the party's platform will increase the chances that some party regime is chosen (over non-delegation), which regime will be chosen is unclear.

6 Conclusion

How do the results of the model match with the introductory descriptions of Carl Albert and Tom Delay that began the paper? Remember, Albert's leadership was characterized as

Table 2: Effects of Changes on the Probability each Regime is Adopted.

Change	Probability Regime is Chosen	
	Carrot Regime	Stick Regime
Adding a Stalwart Legislator	Increases	Increases
Adding a Centrist Legislator	Increases	Increases
Shift a Stalwart to a Centrist	Indeterminate	Indeterminate
Partisan Utility Increases	Increases	Increases
Increase Centrist Legislator Costs	Decreases	Decreases

"inactive and weak." He lead during a time when the Democratic Caucus was split between the Northern and Southern wings of the party. Thus, on any issue there were likely to be a large number of centrists. Furthermore, on many issues Southern Democrats would suffer large electoral costs if they were to vote with the party leadership. In the dual-utility model both of these factors lead to legislators withholding influence from their leaders.

By contrast, Delay—nicknamed "The Hammer"—was described as a capable, almost fearsome, leader. Although his majority was smaller than Albert's, the members of the Republican caucus during his tenure were often described as "ideologues" (Dubose and Reid, 2004) suggesting few centrists, and elevated partisan utility. In the dual-utility model both of these factors lead to legislators empowering their leaders to exert electoral influence.[29]

[29]Although both of these anecdotes are subject to the standard criticism applied to party loyalty scores, (Krehbiel, 1999) they are in general agreement with the dual-utility model.

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Appendix A Some Properties of Order Statistics

This section contains some results about the distribution of order statistics that are used in the paper. The c.d.f. of the k^{th} order statistic (lowest value) from n independent draws from a distribution with c.d.f. $F(v)$ is given by (6) and reprinted here:

$$F_k^n(v) = \sum_{j=0}^{n-k} \left(\begin{array}{c} n \\ j \end{array} \right) F(v)^{n-j}(1-F(v))^j$$

Lemma A.1 *$F_{k+1}^n(v)$ first order stochastic dominates $F_k^n(v)$.*

Proof.

$$\begin{aligned} F_k^n(v) - F_{k+1}^n(v) &= \sum_{j=0}^{n-k} \left(\begin{array}{c} n \\ j \end{array} \right) F(v)^{n-j}(1-F(v))^j - \sum_{j=0}^{n-(k+1)} \left(\begin{array}{c} n \\ j \end{array} \right) F(v)^{n-j}(1-F(v))^j \\ &= \left(\begin{array}{c} n \\ n-k \end{array} \right) F(v)^k(1-F(v))^{n-k} \geq 0 \end{aligned}$$

so $F_k^n(v) \geq F_{k+1}^n(v)$, and thus $F_{k+1}^n(v)$ first order stochastic dominates $F_k^n(v)$. ■

Lemma A.2 *$F_k^n(v)$ first order stochastic dominates $F_k^{n+1}(v)$.*

Proof.

Show

$$F_k^{n+1}(v) - F_k^n(v) = \left(\begin{array}{c} n \\ n+1-k \end{array} \right) F(v)^k(1-F(v))^{n+1-k} \geq 0 \qquad \text{(A.1)}$$

by induction. Consider $F_z^{n+1}(v) - F_z^n(v)$ for $z = n$:

$$\begin{aligned} F_{z=n}^{n+1}(v) - F_{z=n}^n(v) &= F(v)^{n+1} + (n+1)F(v)^n(1-F(v)) - F(v)^n \\ &= nF(v)^n(1-F(v)) = \left(\begin{array}{c} n \\ n+1-z \end{array} \right) F(v)^z(1-F(v))^{n+1-z} \end{aligned}$$

Now, assume (A.1) holds for z and show that it holds for $z-1$:

$$\begin{aligned}
F_{z-1}^{n+1}(v) - F_{z-1}^{n}(v) &= \sum_{j=0}^{n+1-(z-1)} \left(\begin{array}{c} n \\ j \end{array} \right) F(v)^{n-j}(1-F(v))^{j} - \sum_{j=0}^{n+1-z} \left(\begin{array}{c} n \\ j \end{array} \right) F(v)^{n-j}(1-F(v))^{j} \\
&= F_{z-1}^{n+1}(v) - F_{z-1}^{n}(v) + \left(\begin{array}{c} n+1 \\ n+1-(z-1) \end{array} \right) F(v)^{z-1}(1-F(v))^{n+1-(z-1)} \\
&\quad - \left(\begin{array}{c} n \\ n+1-z \end{array} \right) F(v)^{z-1}(1-F(v))^{n+1-z} \\
&= \frac{n!}{(z-1)!(n+1-z)!} F(v)^{z-1}(1-F(v))^{n-1-z} \left[F(v) + \frac{n+1}{n+2-z}(1-F(v)) - 1 \right] \\
&= \left(\begin{array}{c} n \\ n+1-(z-1) \end{array} \right) F(v)^{z-1}(1-F(v))^{n+1-(z-1)}
\end{aligned}$$

as desired. So $F_k^{n+1}(v) \geq F_k^n(v)$, and thus $F_k^n(v)$ first order stochastic dominates $F_k^{n+1}(v)$. Note the inequality in (A.1) is strict for $v \in (0, \overline{v})$. ■

Lemma A.3 *$F_{k+1}^{n+1}(v)$ first order stochastic dominates $F_k^n(v)$.*

Proof.

Show

$$F_k^n(v) - F_{k+1}^{n+1}(v) = \left(\begin{array}{c} n \\ k \end{array} \right) F(v)^{k}(1-F(v))^{n+1-k} \geq 0 \tag{A.2}$$

by induction. Consider $F_z^n(v) - F_{z+1}^{n+1}(v)$ for $z = n$:

$$\begin{aligned}
F_{z=n}^{n}(v) - F_{z+1=n+1}^{n+1}(v) &= F(v)^n - F(v)^{n+1} \\
&= F(v)^n(1-F(v)) = \left(\begin{array}{c} n \\ z \end{array} \right) F(v)^{z}(1-F(v))^{n+1-z}
\end{aligned}$$

Now, assume (A.2) holds for z and show that it holds for $z-1$:

$$\begin{aligned}
F_{z-1}^n(v) - F_z^{n+1}(v) &= \sum_{j=0}^{n-(z-1)} \left(\begin{array}{c} n \\ j \end{array} \right) F(v)^{n-j}(1-F(v))^j - \sum_{j=0}^{n+1-z} \left(\begin{array}{c} n \\ j \end{array} \right) F(v)^{n-j}(1-F(v))^j \\
&= F_{z-1}^n(v) - F_z^{n+1}(v) + \left(\begin{array}{c} n \\ n+1-z \end{array} \right) F(v)^{z-1}(1-F(v))^{n+1-z} \\
&\quad - \left(\begin{array}{c} n+1 \\ n+1-z \end{array} \right) F(v)^{z}(1-F(v))^{n+1-z} \\
&= \frac{n!}{(z-1)!(n-z)!} F(v)^{z-1}(1-F(v))^{n-1-z} \left[\frac{F(v)}{z} + \frac{1}{n+1-z} - \frac{(n+1)F(v)}{(n+1-z)z} \right] \\
&= \left(\begin{array}{c} n \\ z-1 \end{array} \right) F(v)^{z-1}(1-F(v))^{n+1-(z-1)}
\end{aligned}$$

as desired. So $F_k^n(v) \geq F_{k+1}^{n+1}(v)$, and thus $F_{k+1}^{n+1}(v)$ first order stochastic dominates $F_k^n(v)$. Note the inequality in (A.1) is strict for $v \in (0, \overline{v})$. ■

Lemma A.4 *If $f(v)$ has full support on the interval $[0, \overline{v}]$ then $F_k^n(v)$ is strictly increasing in v on $(0, \overline{v})$.*

Proof. If $f(v)$ has full support on the interval $[0, \overline{v}]$, so $F(v)$ is strictly increasing on this interval. The derivative of $F_k^n(v)$ with respect to $F(v)$ is:

$$\begin{aligned}
\frac{dF_k^n(v)}{dF(v)} &= nF(v)^{n-1} + n((n-1)F(v)^{n-2}(1-F(v)) - F(v)^{n-1}) \\
&\quad + \cdots + \frac{n!}{j!(n-j)!}((n-j)F(v)^{n-j-1}(1-F(v))^j - (j)F(v)^{n-j}(1-F(v))^{j-1}) \\
&\quad + \frac{n!}{(j+1)!(n-j-1)!}((n-j-1)F(v)^{n-j-2}(1-F(v))^{j+1} - (j+1)F(v)^{n-j}(1-F(v))^j) \\
&\quad + \cdots + \frac{n!}{k!(n-k)!}((k)F(v)^{k-1}(1-F(v))^{n-k} - (n-k)F(v)^k(1-F(v))^{n-k-1}) \\
&= k \left(\begin{array}{c} n \\ k \end{array} \right) F(v)^{k-1}(1-F(v))^{n-k} > 0 \quad \text{when } F(v) \in (0,1)
\end{aligned}$$

so $F_k^n(v)$ is strictly increasing in $F(v)$ on $(0,1)$ which is strictly increasing in v on $(0, \overline{v})$. Therefore, $F_k^n(v)$ is strictly increasing in v on $(0, \overline{v})$. ■

Lemma A.5 *If a distribution $G(v)$ FOSD a distribution $F(v)$ then the distribution of the k^{th} order statistic (from n draws) $G_k^n(v)$ first order stochastic dominates $F_k^n(v)$:*

$$G(v) \leq F(v)\ \forall v \in [0, \overline{v}] \Rightarrow G_k^n(v) \leq F_k^n(v)\ \forall v \in [0, \overline{v}]$$

Proof. From the definition of $F_k^n(v)$ it should be clear that $F_k^n(v) \equiv H(F(v))$ where $H(\cdot)$ is some function. The proof of Lemma A.4 makes it clear that $H(\cdot)$ is strictly increasing on $(0, 1)$ so $G(v) \leq F(v) \Rightarrow H(G(v)) \leq H(F(v)) \equiv G_k^n(v) \leq F_k^n(v)$ ■